SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Construct a set of orthonormal vectors from the set of vectors (1, 0, 1), (0, 0, 1) and (1, 1, 0).

17. Obtain the Cauchy-Riemann condition to be satisfied by an analytic function. Also test the analyticity of the function $w = \sin z$.

18. Find the characteristic equation of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & 1 \end{pmatrix}$. Using Cayley-Hamilton theorem find A^{-1}.

19. Find finite Fourier sine and cosine transforms of the function $f(x) = x^2$ for $0 < x < 4$.

20. Define Hermite polynomial and derive recurrence formulae:

    (a) $H_n'(x) = 2n\,H_{n-1}(x)$

    (b) $2xH_n(x) - H_{n+1}(x) = H_n'(x)$ and

    (c) $2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$.

NOVEMBER/DECEMBER 2023

23PPH11 — MATHEMATICAL PHYSICS

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. What is linear vector space? Mention any two of its properties.

2. Define isomorphism of vector space.

3. Show that $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$.

4. State Cauchy' s integral theorem.

5. Check the orthogonality of the matrix $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$.

6. Find the adjoint of the matrix $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$.

7. Define Fourier integral transform of a function and state its linearity property.

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8. Find the Laplace transform of Delta function.

9. What is the value of the integral $\int_{-\infty}^{+\infty} f(x)\,\delta(x-4)\,dx$?

10. What is Green function? State its symmetry property.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Show that the vectors (1, 2, –3), (1, 3, –2) and (2, –1, 5) are linearly independent.

Or

(b) Show that for any $v \in U+W$, there exist unique vectors $u \in U$ and $w \in W$ such that $v = u + w$, if and only if $U \cap W = \{0\}$.

12. (a) Write all possible Laurent series for the function $f(Z) = \frac{1}{z(z+2)^3}$ about the pole $z = -2$.

Or

(b) Using residue theorem evaluate $\int_C \frac{1+z}{z(2-z)}\,dz$ where c is the circle $|z| = 1$.

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13. (a) If $A = \begin{pmatrix} 2+i & 3 & -1+3i \\ -5 & i & 4-2i \end{pmatrix}$, show that AA^* is a Hermitian matrix, where A^* is the conjugate transpose of A.

Or

(b) Find the eigen values of the matrix $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$.

14. (a) State and prove convolution theorem in Fourier transform.

Or

(b) Find inverse Laplace transform of $\ln\left(\frac{s^2+w^2}{s^2}\right)$.

15. (a) Using Rodrigue's formula for Legendre polynomial obtain the values $P_2(x)$ and $P_3(x)$.

Or

(b) Find the Green's function for the boundary value problem $\frac{d^2y}{dx^2} - k^2 y = f(x)$ with boundary conditions, $y(\pm\infty) = 0$.